

The Light Bulb Problem

Names of students in your group:

Submission: Each student must submit their answer separately in PDF form to Moodle by the end of the class. No email submissions are allowed. Google how to convert a Google doc to a PDF if you haven't done it before.

The Problem

You have a number of light bulbs, and you need to determine the highest floor in a building from which a bulb may be dropped without breaking. All light bulbs are identical with the following properties:

1. If a light bulb is dropped from a given floor and breaks, then any other light bulb will also break if dropped from that or any higher floor.
2. If a light bulb is dropped from a given floor and does not break, then neither it nor any other light bulb will break if subsequently dropped from that or any lower floor.
3. A light bulb may be dropped any number of times until it breaks, after which it is unusable and cannot be dropped again.

A “drop” consists of taking a single light bulb in the elevator up to a particular floor, dropping the light bulb from that floor, taking the elevator back down, and checking whether the dropped light bulb broke.

Question 1

Given 7 destructible light bulbs and a building with 100 floors, how do you determine the height that the light bulb breaks?

Question 2

Can we do better? That is, can we determine the height that the light bulb breaks, using *fewer than 7 destructible light bulbs* for a building with 100 floors?

Question 3

Given *2 destructible light bulbs*, and a building with 100 floors, how do you determine the height that the light bulb breaks? How many drops does your method require?

Question 4

How can we find the solution using 2 light bulbs for a building with 100 floors using the fewest number of drops? (hint: should be less than 19 drops)

Question 5: Generalization for 2 light bulbs

What is the maximum number of floors that can be tested using n drops and 2 light bulbs?

Question 6: Generalization for m light bulbs

What is the maximum number of floors that can be tested using n drops and m light bulbs?